Computational Analysis of Slot Coating on a Tensioned Web

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When a web (thin substrate with considerable flexibility) travels through a coating applicator in free span between the two supporting rollers, the gap between web and coating applicator is determined primarily by the hydrodynamic stresses in the coating flow and externally applied tension on the web. The elastohydrodynamic interaction between the tensioned-web deformation and the viscous liquid flow in the coating bead has not been extensively studied, in spite of its practical importance. A tensioned-web wrapped around a slot coater was analyzed here by finite-element computations. The externally applied tension on the web typically is much greater than the force due to hydrodynamic stresses. Hence, tension in the web is assumed to be constant in the process direction tangential to the web surface. By neglecting the bending and twisting moments as in the approximation of membrane theory, the shape of the flexible web is determined by a normal stress balance equation, quite similar to that of the free surface of coating flow with a constant surface tension. In this work, elastohydrodynamic effects in tensioned-web slot coating are examined with respect to variations of the liquid feeding rate and externally applied web tension, web wrap angle, and other variables.

Introduction

In the continuous liquid coating process, a liquid layer is deposited through a coating applicator onto a moving solid substrate that is often in the form of flexible web. Positioning the coating applicator against an unsupported web segment has been found to be quite effective in enabling coating of very thin liquid films (Tanaka and Noda, 1984; Shibata and Scriven, 1986; Shibata et al., 1992) and is often referred to as "tensioned-web coating." The gap between the coating applicator and web (the coating gap) is controlled by the elastohydrodynamic interaction between hydrodynamic force due to the liquid flow and elastic force due to the web deformation (for a review, see Pranckh and Coyle, 1997). The formation of a self-regulated coating gap in tensioned-web coating avoids the difficulties associated with mechanically setting a rigid coating gap with respect to a backing roll, especially when the gap is required to be extremely narrow for very thin coatings. In spite of its simplicity and practical importance, serious analyses of tensioned-web coating are scarce in the published literature (Pranckh and Coyle, 1997).

The purpose of the present work is to analyze tensioned-web coating by means of Galerkin finite element computations, using advanced techniques for free-surface flows as summarized by Christodoulou et al. (1997). In addition to dealing with the complexities arising from the deformations of fluid interfaces as in all coating flow computations, the shape of the flexible web as part of the confinement of the liquid flow is also unknown *a priori*. Different levels of sophistication can be applied in mathematically describing the mechanical behavior of a flexible web. In view of the fact that the web in coating operations is typically thin compared with the length of the coating bead and the local deformations of infinitesimal segments remain small, the cylindrical shell theory with linear elastic models (Flügge, 1973) was used by Shibata and Scriven (1986) in a few select case studies of tensioned-web slot coating. The treatment of the web's mechanical behavior in this work is further simplified by adopting the membrane theory where the bending and twisting moments in the shell theory are neglected (Flügge, 1973). The membrane theory is especially applicable to the case where a very thin web is used. Moreover, if the externally applied tension on the web is much greater than the forces due to the hydrodynamic effects of liquid flow, as is often the case in practice,

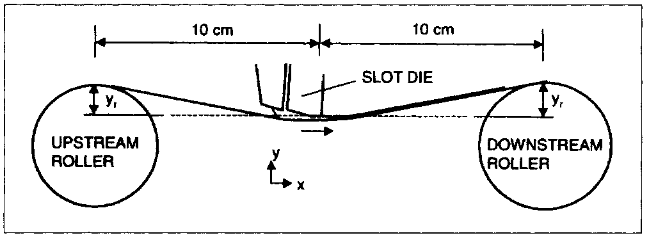

Figure 1. System of slot coating on a tensioned web.

the tension in the web can be assumed to be constant in the process direction tangential to the web surface. Thus, the shape of the flexible web is determined by a normal stress balance equation that is quite similar to that of the gas–liquid interface of coating flow with a constant surface tension. Within the present framework, the theoretical description of tensioned-web coating is made as simple as possible while retaining the essential ingredients of elastohydrodynamic interactions.

Primary attention in this work is paid to the process utilizing a slot die as the coating applicator (Figure 1) because of its practical importance and, to some extent, the reduction of computational complexities in obtaining a converged solution by easily confining the coating bead with solid walls and pinning the downstream meniscus at the die lip edge. Here, elastohydrodynamic effects in tensioned-web slot coating are examined with respect to variations of the liquid feeding rate and externally applied web tension, web wrap angle, and other variables.

Mathematical Description of Web Behavior

In steady state operation, the slot coating flow should ideally not vary across the width of the web. With Newtonian liquid, the flow field is governed by the Navier–Stokes system for two-dimensional, incompressible flow (for detailed formulations, see Christodoulou et al., 1997). Likewise, the mechanical behavior of the web should remain two-dimensional and may be described by the theory of inextensible, thin, cylindrical shells (Pranckh, 1989; Pranckh and Coyle, 1997). The governing equations can then be derived from first principles by appropriately integrating the stresses over the thickness of the shell, which leads to equations of a tangential and a normal force balance, and a moment balance along the shell (Flügge, 1973; Pranckh, 1989). Thus, the web can be treated in the two-dimensional domain as a mathematical curve and only one-dimensional discretization is needed in the finite-element computations. If we consider the fact that the elastohydrodynamic effects in tensioned-web coatings arise primarily from the externally applied tension in the web along the process direction (unlike that in flexible blade coating, which relies on the bending stiffness of the blade), a further simplified membrane theory (Flügge, 1973) may be applied in studying the fundamental aspects.

The membrane theory is arrived at by neglecting the bending and twisting moments in the shell theory. In the cylindrical shell theory for two-dimensional coating analysis, the twisting moment is absent (Pranckh, 1989; Pranckh and Coyle, 1997). The relative significance of the bending moment in comparison with the web tension may be evaluated with a dimensionless number N_{BT} constructed as the bending stiffness $Et^3/[12(1-\upsilon^2)]$ (where E, t, and υ are Young's modulus, thickness, and Poisson's ratio, respectively, of the web material) divided by the product of web tension and the square of the radius of web curvature. For most polymer materials used as coating webs, E and υ are on the order of $10^9\ \mathrm{N \cdot m^{-2}}$ and 0.3, respectively. Typically, the externally applied tension on the web is about $175\ \mathrm{N \cdot m^{-1}}$ [based on 1 lb/in. (18 g/mm)]. Considering the values of typical slot lip length, that is, $1\ \mathrm{m}^{-3}$, and wrap angle of web about the coating applicator, that is, 5°, the radius of web curvature could be estimated as about 2.5×10^{-2} m. The value of N_{BT} therefore should be about 10^{-3} for a web of $t = 100\ \mu$m thick. Because N_{BT} is proportional to t^3, it can decrease by almost an order of magnitude with a reduction of web thickness by a factor of one half. Therefore, the membrane approximation should be valid and accurate for most cases of tensioned-web slot coating, especially for thin webs at high web tension and small wrap angles.

With the approximation of the membrane theory, the local resultant of the internal normal stresses N acting in the direction of the tangent t to the web is governed by

$$\begin{cases} \dfrac{dN}{ds} - \underline{t}\underline{n}:\underline{\underline{T}} = 0 \\ N\nabla\cdot\underline{n} - \underline{n}\underline{n}:\underline{\underline{T}} = 0 \end{cases}, \tag{1}$$

where s is the arc length along the web, $\nabla\cdot\underline{n}$ is the local curvature of the web surface, and $\underline{\underline{T}} \equiv p\underline{\underline{I}} + [\nabla\underline{u} + (\nabla\underline{u})^T]$ is the hydrodynamic stress tensor. Here $\underline{n}$ and $\underline{t}$ denote the local unit normal and tangential vectors to the boundaries. As usual, variables in mathematical formulations herein are dimensionless with length measured in units of the characteristic length L defined as the size of the feed slot, fluid flow velocity $\underline{u}$ in units of the coating process speed U, hydrodynamic stresses $\underline{\underline{T}}$ in units of μUL^{-1} where μ is the viscosity of coating liquid, and the local resultant of the web internal stresses N in units of μU.

If the contribution of viscous shear force due to flow in the coating bead is negligible in comparison with the magnitude of the applied tension on the web (as is verified by computational results shown later on), the first equation in (1) may be disregarded and N may be treated as a constant along the web. Thus, the only unknown corresponding to the web deformation is the local curvature of the web surface, in response to the normal component of hydrodynamic stresses due to the liquid flow. The physical description of web deformation is now the same as that for a gas–liquid interface with a constant surface tension. The dimensionless web tension N associated here with the web deformation infers similar effects as the inverse capillary number associated with the gas–liquid interface deformation in the conventional mathematical formulation for coating flows (Christodoulou et al., 1997). Further simplification could cause loss of theoretical consistency in describing the fundamental elastohydrodynamic effects in tensioned-web coatings.

Computational Treatments

As is commonly adopted in coating flow analyses, the Galerkin finite-element method (see, for example, Christodoulou et al., 1997, and the literature cited therein) is used in the present work for computing the solutions for the nonlinear free-boundary problem of tensioned-web slot coating. The numerical code used here is modified and enhanced with respect to that previously developed for analyzing coating flows with electrostatic effects (Feng and Scriven, 1992, 1993).

The problem domain as shown in Figure 2 is topologically the same as that typically configured for computations of slot-coating flow with rigid substrates (Sartor, 1990; Durst and Wagner, 1997) and is subdivided into a set of quadrilateral elements (Strang and Fix, 1973). On each element, which is mapped onto a unit square in the $\xi\eta$ domain, the fluid velocity field is expressed in an expansion of biquadratic basis functions, whereas the pressure field is expressed in an expansion of linear discontinuous basis functions. To facilitate simultaneous solution of flow field and deformable domain shape, an elliptic mesh generation with subparametric nodal position mapping as developed by Christodoulou and Scriven (1992) and modified by de Santos (1991) is employed here. In the present work, the entire problem domain is divided into 720 elements as tested to be quite sufficient for obtaining mesh-independent numerical solutions. To relieve the nonintegrable singularity at the dynamic wetting line, a dynamic contact angle is prescribed and the fluid is allowed to slip in one element adjacent to the dynamic wetting line along the web surface. With such an *ad-hoc* treatment, as is commonly used in modeling coating flows (Christodoulou and Scriven, 1989; Christodoulou et al., 1997), any inaccuracies in physical description of dynamic wetting are highly localized around the dynamic wetting line, and the macroscopic flow field and the shapes of free surfaces appear to be insensitive to the variation of the slip coefficient (with its nominal value assumed to be 0.05) over orders of magnitude.

With the web behavior described by the membrane theory, the deformable web becomes part of the boundary of the problem domain. The boundary conditions at the deformable web surface are the normal force balance

$$N\nabla\cdot\underline{n} - \underline{n}\,\underline{n}:\underline{\underline{T}} = 0, \tag{2}$$

and the no-slip and no-penetration conditions

$$\underline{t}\cdot\underline{u} = 1 \quad \text{and} \quad \underline{n}\cdot\underline{u} = 0. \tag{3}$$

For those nodes at the web surface, the weighted residual equations, corresponding to x-, y-components of the momentum equation, and an elliptic mesh equation are replaced with the no-slip normal force balance, and no-penetration conditions, respectively.

At the intersection of the outflow boundary and the web, the node is allowed to move freely and the local slope of the web is determined by the position of the downstream roller, such as

$$\frac{dy}{dx} = \frac{y_n - y_r}{x_n - x_r}. \tag{4}$$

Here x_n and y_n describe the unknown nodal position where the web intersects the outflow boundary, and x_r and y_r are the fixed reference position of the web as supported by the downstream roller. At the dynamic wetting line where the web intersects the upstream meniscus, the corresponding node is also free to move and the local slope of the web is determined by the position of the upstream roller in the same fashion as in Eq. 4. Such a treatment of a flexible web is similar to that for a two-dimensional meniscus with specified static contact angles at two ends. When Galerkin's method of weighted residuals is applied to the local curvature term along the web surface, the end condition (Eq. 4) can be easily incorporated into the residual equation as a natural boundary condition in terms of the local web slope. Thus, the well-established numerical techniques (Christodoulou et al., 1997) can be conveniently adapted for studying elastohydrodynamic effects in tensioned-web coating. Here, the system of Navier–Stokes equations, together with Eqs. 2 and 3 due to the deformable web and elliptic mesh generation equations for tessellating the problem domain of an unknown shape, are solved simultaneously. Quadratic convergence to the desired solution is achieved by using the full Newton iteration scheme.

Results

In this work (Figure 1), the web is assumed to span between the two supporting rollers 20 cm apart, moving at a speed of 1 m·s^{-1}. The downstream die slip edge is aligned at the middle of the suspended web. Typically, both upstream and downstream rollers are positioned with $y_r = 5$ mm (Figure 1) so that the web wrap angle is about 5°. Figure 3 shows the geometric details of the slot die considered in the present work, following some guidelines described by Tanaka and Noda (1994). The characteristic length that equals the feed slot is assumed to be 0.1 mm. The surface tension at the gas–liquid interface is fixed at a value of 0.05 N·m^{-1}. The nominal value of the liquid viscosity is taken to be 0.1 N·sm^{-2}. Thus, the nominal values of Reynolds number Re and

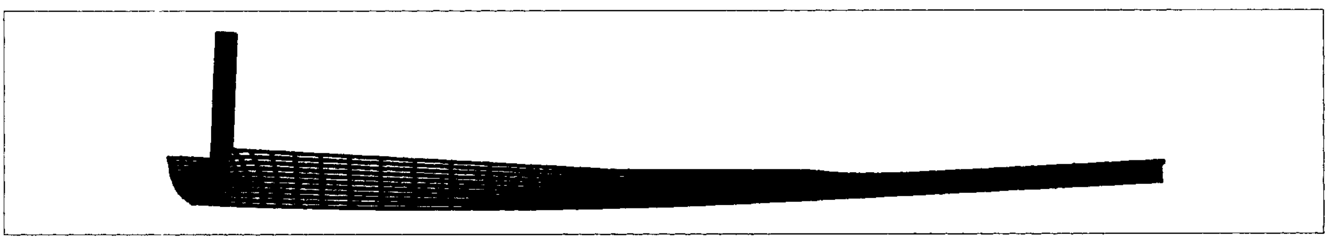

Figure 2. Typical finite-element tessellation in the coating bead region.

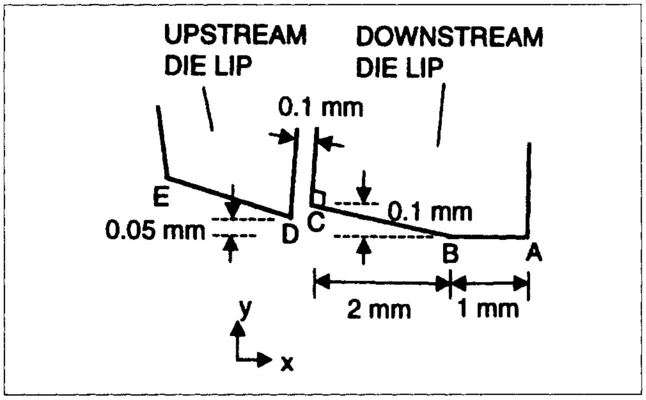

Figure 3. Geometry of the slot die.

Slot die is configured for a general theoretical study only with no implication as an optimal configuration for any practical applications.

capillary number Ca are 1 and 2, respectively (assuming the density of liquid is 10^3 kg·m^{-3}). The nominal value of the web tension is assumed as 175 N·m^{-1} (1 lb/in. = 18 g/mm). The upstream meniscus is allowed to move along the upstream die lip, with a static contact angle of 70° and a dynamic contact angle of 120°. Hence, the location of the upstream meniscus can be adjusted by varying the applied vacuum pressure thereby.

The computed flow field is shown in Figure 4 for a liquid feeding rate of 1.0, corresponding to a coating of final thickness 0.1 mm. With the present (nominal) configuration, the fluid flow is evenly distributed in the coating bead as desired and no vacuum box is needed at the upstream meniscus for keeping the liquid wet appropriate portion of the die lip. The computed viscous shear stress exerted by flow in the coating bead on the web is typically on the order of 1 (in units of $\mu U \cdot L^{-1}$). When integrated over the length of the entire coating bead according to Eq. 1, the variation in the tension on the web due to the fluid flow in the coating bead should be about 1 N·m^{-1}, which is indeed negligible compared with the magnitude of the applied web tension (175 N·m^{-1}), as assumed in the present work; hence, the theoretical framework is self-consistent.

In general, the apparent curvature of a deformable web is relatively small compared with that of a gas–liquid interface because of the relatively large magnitude of the tension typically applied to the web. The computed web curvature may be reduced slightly with increasing web tension, as seen in Figure 5a. Without adjusting the vacuum pressure, the upstream meniscus is pushed further upstream when the web tension is increased. Shown in Figure 5b is the fact that greater tension applied on the web results in higher pressure in the coating bead, as observed previously by Shibata and Scriven (1986) (also see Shibata et al., 1992; Tomaru and Scriven, 1998). The pressure in coating bead corresponding to the web deformation appears to be proportional to the product of web curvature and web tension (as is similar to that of fluid interface with a constant surface tension).

Because the curvature of the web is mostly controlled by the wrap angle, varying the roller position of y_r (Figure 1) should change the general magnitude of pressure in coating bead and the upstream meniscus position. The wrap angle effects on the pressure magnitude and upstream meniscus position are illustrated in Figures 6a and 6b. With appropriately adjusted wrap angle according to tension applied on the web, the upstream meniscus can be maintained at a fixed position without using a vacuum box. Only the downstream meniscus is shifted slightly in response to the wrap angle variation at different magnitudes of tension applied on the web. As indicated in Figure 6, the fixed position of upstream meniscus corresponds to approximately the same overall magnitude of pressure in the coating bead despite the variation in web tension. It is therefore desirable to use a spring-mounted slot die or spring-mounted web-supporting rollers for coating on a tensioned web. The system can therefore automatically adjust the wrap angle based on the pressure exerted by the web at various magnitudes of applied tension and the upstream meniscus can be maintained at an almost fixed position without using a vacuum box.

A major advantage of the tensioned-web slot coating over standard slot coating with a rigid backing roll is self-regulation of the coating gap (that is, the clearance between the die lip and web) according the liquid feeding rate. To obtain a successful slot coating, the ratio of coating thickness and coating gap must be greater than a critical value (Sartor, 1990; Durst and Wagner, 1997; Carvalho, 1998). For small capillary number cases, the critical value for the ratio of coating thickness and coating gap was found by Higgins (1980) to be equal to 0.67 $Ca^{2/3}$ on the basis of an approximate analysis using a formula of Ruschak (1976) as an extension of the Landau-Levich (Landau and Levich, 1942) film coating theory. As shown in Figure 7 over a wide range of liquid feeding rates,

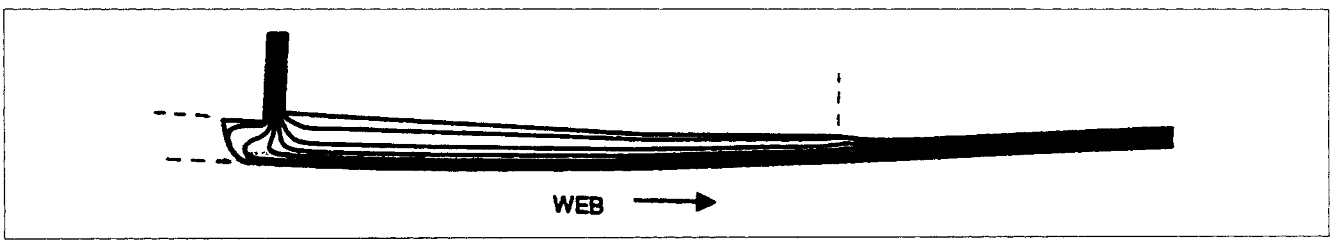

Figure 4. Slot coating on a tensioned web.

Streamlines for the nominal case where the vacuum pressure at upstream meniscus is set to zero. Streamline values chosen are 0.01, 0.1, 0.2, 0.3, 0.4, 0.49.

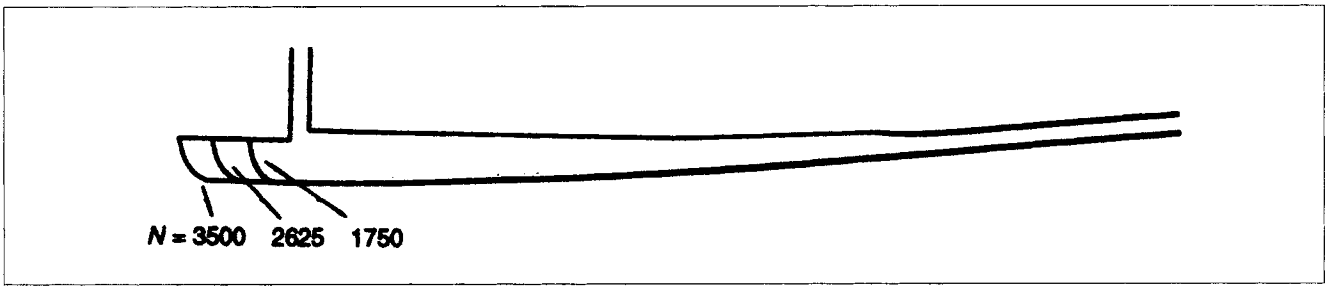

Figure 5a. Web tension effects in tensioned-web slot coating: overall coating bead shape variation.
Downstream and upstream y_r are both set to 5 mm. The vacuum pressure at upstream meniscus is set to zero.

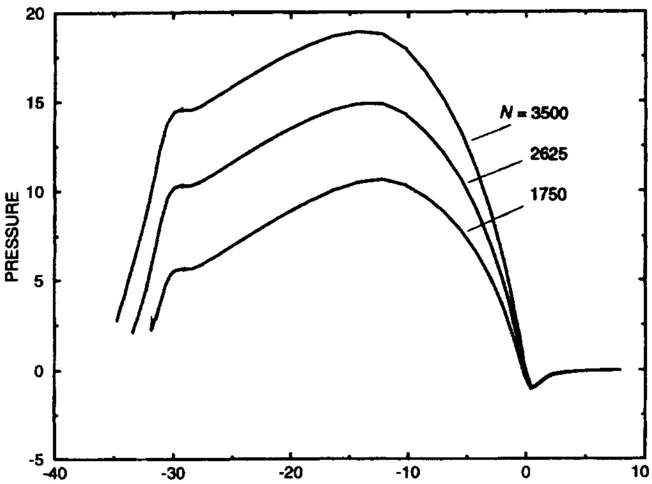

Figure 5b. Pressure distribution in coating bead, corresponding to web tension at 1,750, 2,625, and 3,500 in Figure 5a.

the gap between the downstream lip and web at the die outlet is always maintained at less than twice the final coating thickness by virtue of the elastohydrodynamic effects. Thus, various coating thicknesses can be obtained by using the same slot die setup without needing to set a small gap between the die lip and web with tight tolerances.

However, Figure 7 also indicates the need of a vacuum box at the upstream meniscus to stabilize the coating bead as the liquid feeding rate is reduced significantly. Stronger vacuum pressure is needed for lower liquid feeding rate corresponding to a thinner coating. If one takes the liberty to vary the orientation of the slot die relative to the web, the need of a vacuum box may be avoided. For example, the flow field shown in Figure 8 is obtained for liquid feeding rate 0.5 by just increasing the downstream y_r from 5 mm to 21 mm while other parameters retain the same values as those in Figure 7. The upstream meniscus for the case of liquid feeding rate 0.5 can stay at virtually the same place as that for the case of liquid feeding rate 1.0 with zero vacuum pressure being ap-

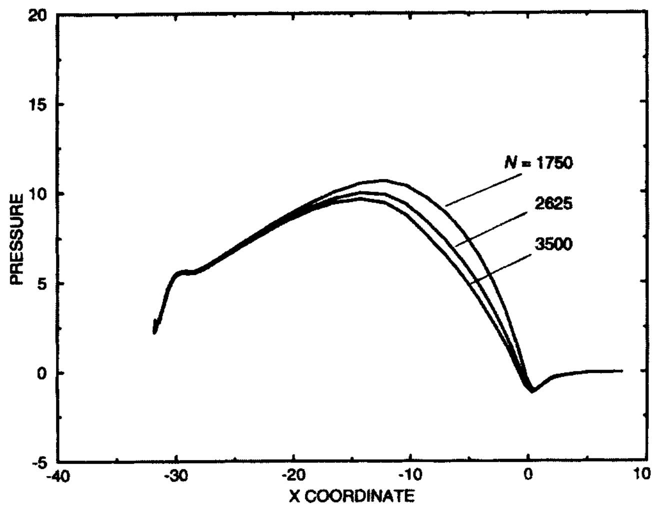

Figure 6a. Web tension effects in tensioned-web slot coating: pressure distribution in coating bead.
Downstream and upstream y_r are both set to 5, 3, and 2.1 mm for web tension at 1,750, 2,625, and 3,500, respectively. The vacuum pressure at upstream meniscus is set to zero.

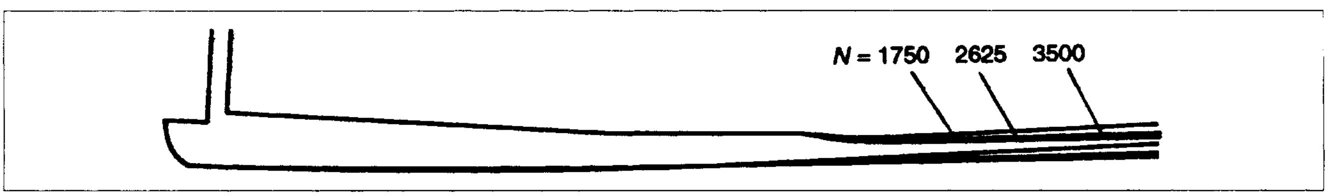

Figure 6b. Overall coating bead shape as web tension varies corresponding to Figure 6a.

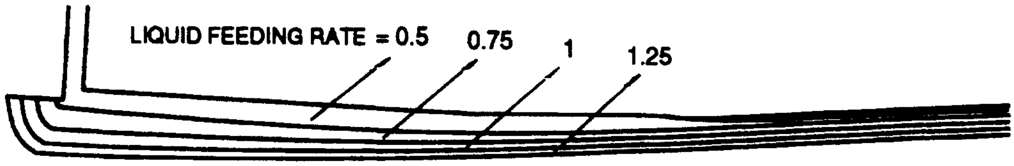

Figure 7. Self-regulation of coating gap in tensioned-web slot coating.

Downstream and upstream y_r (Figure 1) are both set to 5 mm, corresponding to liquid feeding rate 0.5, 0.75, 1, 1.25 with respect to that of the nominal value. The corresponding values of applied vacuum pressure at the upstream meniscus are 35, 5, 0, and −2, respectively, to keep the liquid wetting appropriate portion of the upstream die lip as shown. The web tension is set at 1,750.

plied by simply raising the downstream roller position relative to the slot die (Figure 1). The drawback here with this particular setup is the formation of an undesirable recirculation vortex in the coating bead as shown in Figure 8. If the slot die is rebuilt so that the distance (in the x direction) between C and B of the downstream die lip (Figure 3) is shortened from 2 mm to 1 mm without changing the relative slope of the line and the y-coordinate of point C subsequently becomes the same as that of D, the recirculation vortex in the coating bead can be eliminated as shown in Figure 9 for the case of liquid feeding rate 0.5 with zero vacuum pressure applied at the upstream meniscus. Therefore, varying the shape of slot die lips and overall geometric configuration of the coating die with respect to the web-supporting rollers can significantly improve the operation quality for tensioned-web slot coating.

Conclusion

A theoretical analysis of slot coating on a tensioned web is carried out in the present work by using the Galerkin finite-element method. To minimize the computational complexities yet still retain the essential ingredients in elastohydrodynamic interactions in tensioned-web coating, web behavior is described by a membrane theory in which the bending and twisting moments in the traditional shell theory are neglected. A dimensional analysis with typical parameter values indicates that the membrane approximation should be valid and accurate for most cases of tensioned-web slot coating, especially for thin webs at high web tension and small wrap angles.

Because the magnitude of tension applied on the web is relatively large, the curvature of deformable web is rather small compared with that of the coating bead menisci. For the same wrap angle, higher pressure is induced in the coating bead with increasing tension applied on the web. As a consequence, the upstream meniscus is pushed further upstream if the external pressure is not varied by using a vacuum box, as observed previously by Shibata and Scriven (1986) (also see Shibata et al., 1992; Tomaru and Scriven, 1998). The present analysis shows that the position of the upstream meniscus can become virtually unaltered as the web tension is varied, when the slot die or the web-supporting rollers are spring-mounted so that the web wrap angle changes according to the tension applied on the web while the pressure in coating bead is maintained at a constant level.

As a major advantage of the tensioned-web coating, the gap between the slot die and web is self-regulated according to the liquid feeding rate, or equivalently the final coating thickness. The feature of self-regulation of coating gap is indeed revealed by the computations performed in the present work. If the die lip configuration is not set up appropriately, however, an undesirable recirculation vortex may form in the coating bead and a vacuum box may be needed to hold the upstream meniscus in position, especially when the liquid feeding rate is relatively low for obtaining very thin coatings. It becomes clear that a careful design of the shape of slot die lips and overall geometric configuration of the coating die with respect to the web-supporting rollers is required to significantly improve the operation quality for tensioned-web slot coating. Fortunately, the large number of degrees of freedom in varying the slot die shape, web tension, wrap angle, and so forth should enable such an endeavor.

It should be noticed that the two-dimensional steady states of coating flows, as analyzed in the present work, may become unstable to two- and three-dimensional disturbances in realistic coating operations. If unstable, a steady-state coating flow is unrealizable in practice. A comprehensive stability analysis may require computations of fully transient solutions of the complete system of nonlinear equations. More often

Figure 8. Slot coating on a tensioned web.

Streamlines for the case of liquid feeding rate 0.5, with zero vacuum pressure applied at upstream meniscus and downstream and upstream y_r (Figure 1) set to 21 and 5 mm, respectively. Streamline values chosen are 0.01, 0.05, 0.1, 0.15, 0.20, 0.25, 0.27, 0.29, and 0.30. The web tension is set at 1,750.

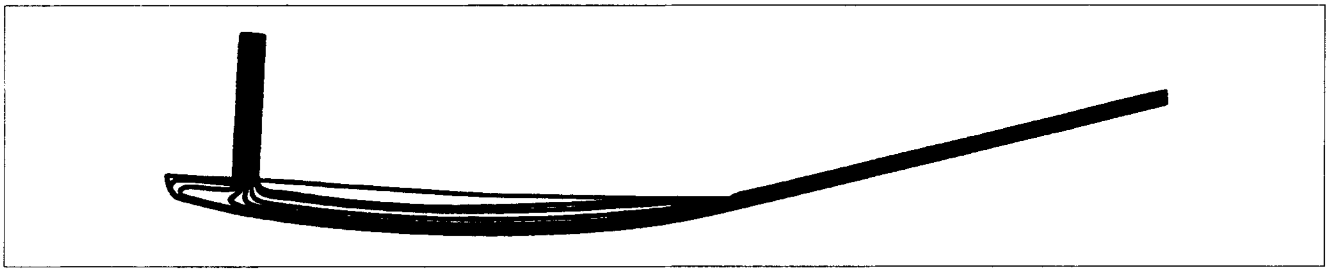

Figure 9. Slot coating on a tensioned web.

As for Figure 8 but for a slot die with shortened downstream lip (the distance in the x direction) between C and B of the downstream die lip (Figure 3) is shortened from 2 mm to 1 mm without changing the relative slope of the line, and the y-coordinate of point C subsequently becomes the same as that of D and both downstream and upstream y_r (Figure 1) are set to 25 mm. Streamline values are 0.01, 0.05, 0.1, 0.15, 0.20, 0.24, and 0.25.

than not, however, an asymptotic analysis with respect to infinitesimally small, two- and three-dimensional disturbances is adequate to predict the critical values of parameters at the onset of coating flow instabilities (Christodoulou et al., 1997). Yet, even simpler consideration based on the knowledge about pressure profile of flow in a converging or diverging channel may provide important insight into the stability of tensioned-web slot coating. For example, the wrapping effect of web about the slot die often leads to formation of a converging channel along the downstream slot lip, as shown in many results herein. The converging channel flow along the downstream slot lip has the same configuration as in knife coating with a converging knife tip that has been known to stabilize the downstream coating meniscus against ribbing (cf. Coyle, 1997). But ribbing instability is not the only cause of coating defects. Because of the lack of solid constraints in tensioned-web coating, the web itself can vibrate freely and, as the web becomes part of the coating device, any nonuniformity, tension variation, and vibration in the web can directly translate into poor coating quality (Pranckh and Coyle, 1997). To fully understand the intricate elastohydrodynamic interactions between the fluid flow and the flexible web in tensioned-web coating, more detailed analyses of stability and sensitivity of a desired two-dimensional steady coating flow against various disturbances, especially those arising from the flexible web, should be conducted. The steady flow analysis presented here with the web behavior described by the membrane theory is expected to facilitate further systematic investigations of tensioned-web coating.

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